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HOMOGENIZATION BASED MODELLING OF ARTERIAL WALL MECHANICS

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Abstract: *A new approach to arterial wall modelling is discussed, which should take into account the mesoscopic and microscopic geometrical arrangement of the tissue, involving elongating smooth muscle cells placed in the extracellular passive matrix. Assuming existence of a representative tissue cell at the mesoscopic scale, the two scale method of homogenization is applied to derive the effective material behaviour at the macroscopic level. Numerical examples are introduced.*

Key words: *artery, homogenization, large deformation, hierarchical modelling*

1. INTRODUCTION

The arterial wall mechanics has been a subject of intensive research over the years and it presents a very complex system in the sense of its material structure and load conditions, which influence the growth and remodelling processes in the whole cardiovascular system. There are many phenomenological material models which attempt to describe the tissue behaviour at macroscopic scale; most of them became outclassed and the only relevant ones are defined in terms of higher order strain invariants which reflect fibrous character of the tissue and some kinds of interactions [2, 4]. In these the deformation energy is a superposition of its contributions which are associated with the particular tissue components: the extracellular tissues (matrix), muscle fibres constituted by chains of muscle cells and other fibrous structures comprising elasti and collagen. Such an approach takes into account the tissue anisotropy as well as its basic features, nevertheless, the resulting mathematical model is still well suited for numerical treatment. However, it is evident that many of the apparent "material" nonlinearities observed at the global macroscopic level are originated in geometrical nonlinearities and interactions in the large deforming "microstructure", .i.e. at the cellular level. Such phenomena can be captured by the multiscale modelling which reflects a hierarchical arrangement of the medium in question. In tissues such a hierarchy spans whole scale of magnification, from nano-structures relevant to filaments and fibrils, over the cellular level up to the global level associated with mechanics of complex organs. The two scale method reported in this paper is one of the most challenging alternatives to this sort of modelling.

We suggest an approach [6, 7] which should take into account the mesoscopic and microscopic geometrical arrangement of the tissue, involving elongating smooth muscle cells placed in the extracellular passive matrix stiffened by collagen fibres. Assuming existence of a representative tissue cell at the mesoscopic scale, we apply the two scale method of homogenization

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to derive the effective material behaviour at the macroscopic level. In particular we focus on medium muscular arteries.

The model of the arterial wall at the mesoscopic scale is designed as a periodic lattice formed by repetition of the reference cell which comprises the matrix and the incompressible inclusion; this should resemble the structure of muscular media. We extend the previous studies of the homogenization method applied in the context of soft tissues [7, 6]. Although here we focus on the hyperelastic materials, which nevertheless can be used to describe instantaneous response, the approach can be extended to the viscoelasticity for describing creep and relaxation processes. [7, 8].

Considering a homogeneous axi-symmetric layout of the vessel and different representative cells, we compute macroscopic responses of the basic load modes, inflation and torsion. The representative cell is constituted by one or more fibre stiffened incompressible inclusions embedded in a hyperelastic matrix; it reflects an idealised model of the smooth muscle cell treated as the tensigrity structure. The aim of the numerical examples is to demonstrate, how the geometrical nonlinearity at the mesoscopic scale gives rise to the apparent material nonlinearity at the macroscopic scale.

1.1. Arterial wall

Arterial wall consists of three layers which in their thicknesses and constitution depend on the particular type of the artery. For the medium arteries the media layer, which contains the muscle cells, is dominant in the mechanical respect, see Fig.1, being bonded by elastic lamelae on its innera and outer surfaces. The innermost and outermost layers, the intima and the adventitia, respectively, are less important from the point of the arterial stiffness and strength, however, they perform other physiological functions. The muscle cells form spiral chains winding over the co-axial cyllindrical surfaces; when contracting they enable to change the lumen of the vessel.

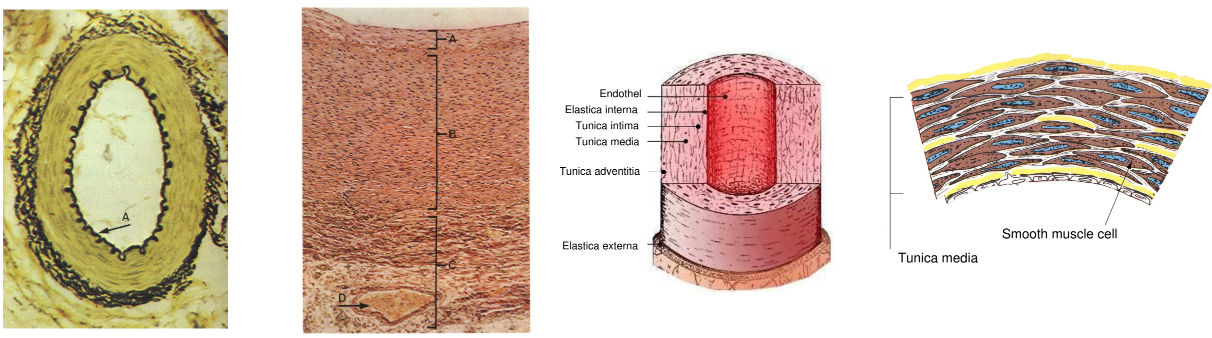

Fig. 1. Layers in the arterial wall.

1.2. Mathematical model

We shall now explain the main features of the two scale model which is based on the hyperelastic material distributed in large deforming micrstructure (from now on we use the "microstructure" as the scale relevant at the level of muscle cells). The periodic heterogeneities at the microscopic level are defined in terms of the reference cell which resembles arrangement of the smooth muscle cell (SMC) embedded in the extracellular matrix (ECM). The cytoskeleton may be considered as a truss-like structure which are fitted to the surface of the inclusion corresponding tho SMC. Such a device allows for modelling of muscular contraction which leads to the contraction of the tissue, as observed at the global macroscopic level. In the real tissue the contractile mechanism is initiated at the nano-structure level by sliding couples of actin and myosin filaments inside the muscle cells; in the mathematical abstraction considered here such mechanism is represented by shortening of the bars incorporated into the trusses.

Although the two scale modelling can be applied for general modes of large deformation, we shall confine ourselves to simplified geometrical assumptions on the arterial wall. We consider only an axisymmetric cyllindrical thick-walled vessels in such load conditions that the structure remains axisymmetric in the course the eformation process.

2. TWO-SCALE METHOD OF HOMOGENIZATION

The two-scale homogenization method [1] is applied to the linearized equations of the incremental formulation problem of large deformation.

2.1. Equilibrium equation

In the *updated Lagrangean framework* the equilibrium equation reads as (cf. [5, 6])

$$\int_\Omega \mathcal{L}\tau_{kl}(\Delta u)\, e_{ij}(v)\frac{1}{J}\,\mathrm{d}x + \int_\Omega \tau_{ij}\,\delta\eta_{ij}(\Delta u; v)\frac{1}{J}\,\mathrm{d}x = \\ = L(v) - \int_\Omega \tau_{ij}\, e_{ij}(v)\frac{1}{J}\,\mathrm{d}x, \quad \forall v \in V_0(\Omega), \tag{1}$$

with $V_0(\Omega) \equiv \{v \in [W^{1,2}(Y)]^n |\, v = 0 \,\mathrm{na}\, \partial\Omega_D\}$,

where $e_{ij}(u) = \frac{1}{2}(\partial_j u_i + \partial_i u_j)$, $\delta\eta_{ij}(u; v) = \frac{1}{2}(\partial_i u_k \partial_j v_k + \partial_i v_k \partial_j u_k)$ with ∂_j being the j-th component of the gradient w.r.t. the deformed configuration associated with the spatial domain Ω, $\mathcal{L}\tau_{ij}(u)$ is the Lie derivative of the Kirchhoff stress τ_{ij} and $L(v)$ is the virtual work of all external forces.

The hyperelastic neo-Hookean law is assumed so that

$$\boldsymbol{\tau} = J\boldsymbol{I}\,p + \mu J^{-2/3}\, dev\, \boldsymbol{b}, \tag{2}$$

where $J = det\,\boldsymbol{F}$, $\boldsymbol{b} = \boldsymbol{F}\boldsymbol{F}^T$ ($\boldsymbol{F}$ is is the deformation gradient), μ is the shear stiffness and p is the hydrostatic pressure. If the material is compressible, the pressure is given by the following relationship which depends on the "volumetric stiffness" γ:

$$p = -\gamma(J - 1). \tag{3}$$

2.2. Homogenized model

In this paragraph we summarize the equations employed the two scale model modelling for the heterogeneous hyperelastic material with incompressible inclusions, see [5, 6]. For those readers, who are not familiar with the homogenization, we outine briefly the main points of modelling

We consider two scales, the macro-scopic one, associated with the global coordianates x, and the micro-scopic one, associated with the y coordinates. For finite scale of heterogeneities, these scales are related each other by the scale parameter

$$y = x/\varepsilon, \quad \varepsilon > 0 .$$

Thus, in our application, x is related with the vessel of artery, whereas y is relevant at scale of individual muscle cells, so that $\varepsilon \approx 100[\mu m]/10[mm] = 10^{-2}$. Further, we assume that at the microscopic level the heterogeneous structure is locally periodic, being formed as a periodic array by copies of the single representative cell Y; here we consider the decomposition

$$Y = Y_m \cup T \cup \partial T \quad T \cap Y_m = \emptyset ,$$

where Y_m corresponds to the ECM substance (the matrix), while T evokes distribution of SMCs (so called inclusions). In the process of homogenization we let $\varepsilon \to 0_+$, so that we obtain the so called limit problem, chracterized by an infinite gap between the two scales. The resulting model, however, is applicable for real situations with $\varepsilon > 0$, as discussed above. As the main advantage of such multiple scale treatment, the “effective” material parameters relevant to the macroscopic scale inherit the specific geometrical arrangement of different constituents defining the microscopic layout. It should be remarked that such “geometrical”feature is missing in the alternative phenomenological, ad hoc macroscopic models.

The material coefficients are defined in the microscopic domain Y according to the decomposition:

$$\mu(y) = \begin{cases} \mu, & x \in Y_m \\ 0, & x \in T \end{cases}, \qquad \gamma(y) = \begin{cases} \gamma, & x \in Y_m \\ \gamma_\infty \to +\infty, & x \in T. \end{cases} \tag{4}$$

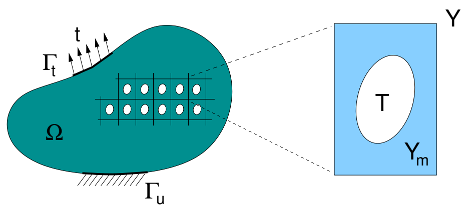

Fig. 2. Macroscpic and microscopic domains.

The two scale model involves the microscopic problems for the corrector basis functions $\boldsymbol{\chi}$, $\boldsymbol{\pi}$ and $\bar{\boldsymbol{\pi}}$, which must be defined for a.a. $x \in \Omega$ by solving the boundary value probles which are now defined.

Microscopic problem: For fixed $x \in \Omega$ and local (deformed) reference microscopic configurationa asotiated with the domain $Y(x)$, find $\boldsymbol{\chi} \in H_\#(Y)$, $\boldsymbol{\pi} \in L^2(Y)$, $\bar{\boldsymbol{\pi}} \in I\!R$ such that $(r, s = 1, 2, (3))$:

$$a_{Y_m}(\chi^{rs} - \Pi^{rs},\, w) - (\pi^{rs},\, div_y w)_{Y_m} + (\bar{\pi}^{rs},\, div_y w)_{Y_m} = 0, \quad \forall w \in H_\#(Y), \tag{5}$$

$$\frac{1}{\gamma}\left(\frac{1}{J}\pi^{rs},\, q\right)_{Y_m} + (q,\, div_y \chi^{rs})_{Y_m} - (q,\, div_y \Pi^{rs})_{Y_m} = 0, \quad \forall q \in L^2(Y), \tag{6}$$

$$(1,\, div_y \chi^{rs})_{Y_m} = -|T|\delta_{rs} \tag{7}$$

where $\Pi_i^{rs} \equiv y_s \delta_{ri}$, $H_\#(Y) \equiv \{v \in [W^{1,2}(Y)]^n |\, v \,\mathrm{je}\, Y - \mathrm{periodic}, \int_Y v(y)\,\mathrm{d}y = 0\}$ is the space of admissible displacement and the bilinear forms $a_Y(u, v)$ and $b_Y(u, v)$ are defined as follows:

$$a_{Y_m}(u,v) = \int_{Y_m} \left(D^{tTK}_{ijkl} + J\bar{p}^0\delta_{ij}\delta_{kl}\right) e^y_{kl}(u)\, e^y_{ij}(v)\tfrac{1}{J}\,\mathrm{d}y, + \int_{Y_m} \left(\tau_{ij}\delta_{kl} - J\bar{p}^0\delta_{kj}\delta_{li}\right) \partial^y_i u_k\, \partial^y_j v_l \tfrac{1}{J}\,\mathrm{d}y. \tag{8}$$

Macroscopic problem: Define the symmetric homogenized stiffness tensor $\hat{\mathcal{Q}}_{ijkl}$ (it holds that $ij\,kl = kl\,ij$, but $ij\,kl \neq ji\,kl$)

$$\hat{\mathcal{Q}}_{ijkl} \equiv \frac{1}{|Y|}\left[a_{Y_m}\left(\Pi^{kl} - \chi^{kl},\ \Pi^{ij} - \chi^{ij}\right) + \frac{1}{\gamma}\left(\frac{1}{J}\pi^{ij},\ \pi^{kl}\right)_{Y_m}\right] + \bar{p}^0(\delta_{kl}\delta_{il} - \delta_{ij}\delta_{kl}). \tag{9}$$

Then, given the average stress in $\hat{\mathcal{S}}_{ij} = \frac{1}{|Y|}\int_Y \tau_{ij} J^{-1}\,\mathrm{d}y$ in Ω, compute the macroscopic displacements $\Delta u^0 \in V(\Omega)$ so that

$$\int_\Omega \hat{\mathcal{Q}}_{ijkl}\,\partial^x_l \Delta u^0_k\, \partial^x_j v_i\,\mathrm{d}x = L(v) - \int_\Omega \hat{\mathcal{S}}_{ij}\, e^x_{ij}(v)\,\mathrm{d}x, \quad \forall v = V_0(\Omega). \tag{10}$$

The above problem is the single step in an incremental algorithm. The homogenized coefficients $\hat{\mathcal{Q}}_{ijkl}$ as well as $\hat{\mathcal{S}}_{ij}$ must be recovered almoste everywhere in Ω, which in means at each Gauss integration point of the finite element approximation of (10). In contrast with linear deformation models, this is the crucial point of whole coupled micro-macro computing in our nonlinear case; after computing the macroscopic strain field, the local microscopic configurations must be updated to recompute the corrector fuctions by solving (5)-(7), thus individually for each Gauss integration point in Ω wherein the "macro-strains" vary nonuniformly. In [6] the method of approximation for $\hat{\mathcal{Q}}_{ijkl}$ and $\hat{\mathcal{S}}_{ij}$ was suggested which is aimed at reducing the number of the microscopic problems to be solved.

We remark that the corrector basis functions multiplied by the macroscopic deformation yield the local "microscopic" perturbation of the macroscopic displacement field. Consequently at the microscopic scale the deformation in $Y(x)$ is non-uniform, as well as the pressure field:

$$\partial_j \Delta u_i(x,y) = \left(\Pi^{rs}_i - \chi^{rs}_i(x,y)\right) \partial^x_s \Delta u^0_r(x), \tag{11}$$

$$\Delta p^0(x,y) = -\pi^{rs}(x,y)\, \partial^x_s \Delta u^0_r(x), \tag{12}$$

$$\Delta \bar{p}^0(x) = -\bar{\pi}^{rs}(x)\, \partial^x_s \Delta u^0_r(x). \tag{13}$$

3. FIBROUS STRUCTURES

3.1. Elastin lamelae

The elastin lamelae which bound the inner and outer surfaces of the media, but which are not distributed inside the media, can be treated as elastic fibres. These are introduced directly in the macroscopic problem formulation. Recalling the assumed cyllindrical topology of the media, they are to be defined at each point of the inner and outer bounding surfaces in terms of the unit vectors aligned with two spiral preferential directions (corresponding tho their most abundant distribution), see Fig.3.

The Cauchy stress in the r-th fibre direction is given by

$$^{[r]}\sigma^{fib}_{ij} = \sigma^{[r]}\nu^{[r]}_i \nu^{[r]}_j, \tag{14}$$

where σ the scalar value of Cauchy stress in the fibre. The directional fibre vector, denoted by $\bar{\boldsymbol{\nu}}$ in its initial configuration, is being updated throughout the deformation proces using the deformation gradient $\boldsymbol{F}$, as follows:

$$\lambda_{[r]}\nu_i^{[r]} = F_{ij}\bar{\nu}_j^{[r]}. \tag{15}$$

The stretch of the fibre, λ, is given as

$$\lambda_{[r]}^2 = \delta_{ki}\, F_{ij}\bar{\nu}_j^{[r]}\, F_{kl}\bar{\nu}_l^{[r]} = \left(\bar{\boldsymbol{\nu}}^{[r]}\right)^T \boldsymbol{C}\,\bar{\boldsymbol{\nu}}^{[r]}, \tag{16}$$

where $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$ is the right Cauch–Green deformaton tensor.

We shall now employ the above relationships to express the Cauchy stress in terms of the 2nd Piola–irchhof stress S, here represented by its magnitude associated with the fibre:

$$^{[r]}\sigma_{ij}^{fib} = \sigma^{[r]}\nu_i^{[r]}\nu_j^{[r]} = \frac{1}{J}\,\lambda_{[r]}^2\,\nu_i^{[r]}\nu_j^{[r]}\,S(\lambda_{[r]}^2 - 1). \tag{17}$$

The tension S is defined using the strain energy function as follows (we abbreviate $\theta = \lambda^2 - 1$)

$$S(\theta) = 2\,\frac{\partial W(\theta)}{\partial\theta}. \tag{18}$$

In the present model strain energy function is defined picewise, using the cubic polynomial

$$W(\theta) = \begin{cases} \frac{1}{6}E\,\theta^3, & \text{for } \theta \geq 0 \\ 0, & \text{for } \theta < 0 \end{cases}, \tag{19}$$

which means no resistance in compression.

The total stress contribution transmitted by all fibres is obtained by summation oer all fibres

$$\sigma_{ij}^{fib} = \sum_r {}^{[r]}\sigma_{ij}^{fib} = \sum_r \frac{1}{J}\,2\,\lambda_{[r]}^2\,\frac{\partial W(\lambda_{[r]}^2 - 1)}{\partial(\lambda_{[r]}^2)}\,\nu_i^{[r]}\nu_j^{[r]}. \tag{20}$$

In analogy, the individual contributions of the stiffnesses yield the superimposed extra stiffness of the homogenized material:

$$D_{ijkl}^{fib} = \sum_r 4\,\lambda_{[r]}^4\,\frac{\partial^2 W(\lambda_{[r]}^2 - 1)}{\partial(\lambda_{[r]}^2)^2}\,\nu_i^{[r]}\nu_j^{[r]}\nu_k^{[r]}\nu_l^{[r]}. \tag{21}$$

For numerical solution by the FEM the fibres are defined in the mesh nodes in terms of the preferential direction vectors, as illustrated in Fig.3.

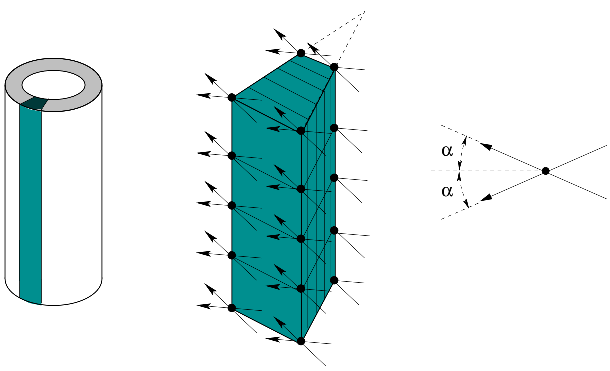

Fig. 3. Fibres defined in the FE mesh points.

3.2. Tensile fibres of the cytoskeleton

In the numerical example discussed in Section 4 we use an extension of the model (5)-(7) which then involves contraction fibres to account for the role of the cytoskeleton in microscopic smooth muscle cells.

For modelling the cytoskeleton fibres we suppose that there is a finite section of the fibre of interest along which the fibre has no mechanical interaction with the surrounding "compact" material. Such situation holds in the intracellular space, assuming sparse fibrous network embedded in the intracellular fluid. Any fibre is allowed to transmit a load only when stratified, as proposed in (19). In this situation each fibre can be assumed to undergo a uniform deformation which is determined by the positions of the fibre's ends y^A and y^B. We shall now record the result of [8], where the contribution of one straight fibre with a finite length to the microscopic equation (5) was derived. Below the quantities labeled by $(\cdot)^{,A}$ and $(\cdot)^{,B}$ refer to their respective values attained at the two end-points. Thus, a single bar of the cytoskeleton results in the additional stiffness tensor A_{ik}

$$A_{ik} = \frac{1}{l_{AB}}\, 4\lambda^4 \frac{\partial^2 W^{\mathrm{a}}(\lambda,\alpha)}{\partial(\lambda^2)^2}\, \nu_i \nu_k + \frac{1}{l_{AB}}\, 2\lambda^2 \frac{\partial W^{\mathrm{a}}(\lambda,\alpha)}{\partial(\lambda^2)}\, \delta_{ik}\;, \tag{22}$$

which is defined using the strain energy function $W^{\mathrm{a}}(\lambda,\alpha)$ depending on the stretch λ of the bar y^A, y^B (with the length l_{AB}) and on the activation parameter α. Then the extended equailibrium equation of the microscopic problem, (5), reads as

$$\begin{aligned} & a_{Y_m}(\chi^{rs} - \Pi^{rs},\, v) - (\pi^{rs},\, div_y w)_{Y_m} + (\bar{\pi}^{rs},\, div_y w)_{Y_m} \\ & \quad + A_{ik}\left(\chi_k^{rs,B} - \Pi_k^{rs,B} - \chi_k^{rs,A} + \Pi_k^{rs,A}\right)\left(v_i^B - v_i^A\right) \;\; = 0, \;\; \forall v \in H_{\#}(Y_m). \end{aligned} \tag{23}$$

4. NUMERICAL EXAMPLE

At the macroscopic level, due to the axisymmetric structure of the vessel, we model only the small part of the vessel wall, which is loaded on the inner surface by the hydrostatic pressure. The microscopic reference cell with several cytoskeleton fibres is shown in Fig.4.

The resulting macroscopic deformations are depicted in Fig.5 a) and b). Two situations are compared: a) structure without inner and outer elastic fibres and without cytoskeleton fibres on the microscopic level; b) inner and outer elastic fibres and cytosceleton fibres are present. In Fig.5 c), d) and e) the microscopic deformation, pressure and stress in a particular point of the microstructure are demonstrated.

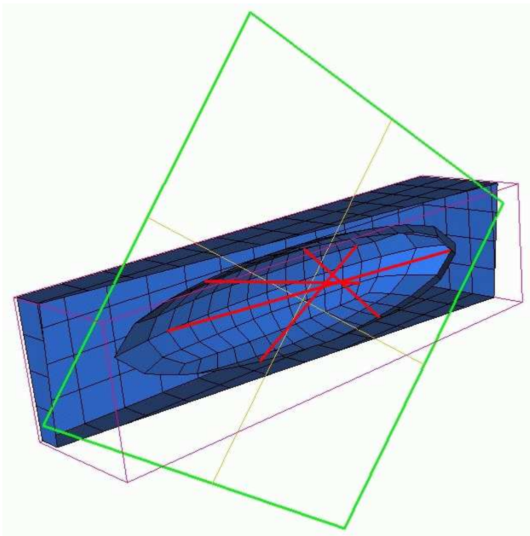

Fig. 4. Reference cell – the cytoskeleton fibers.

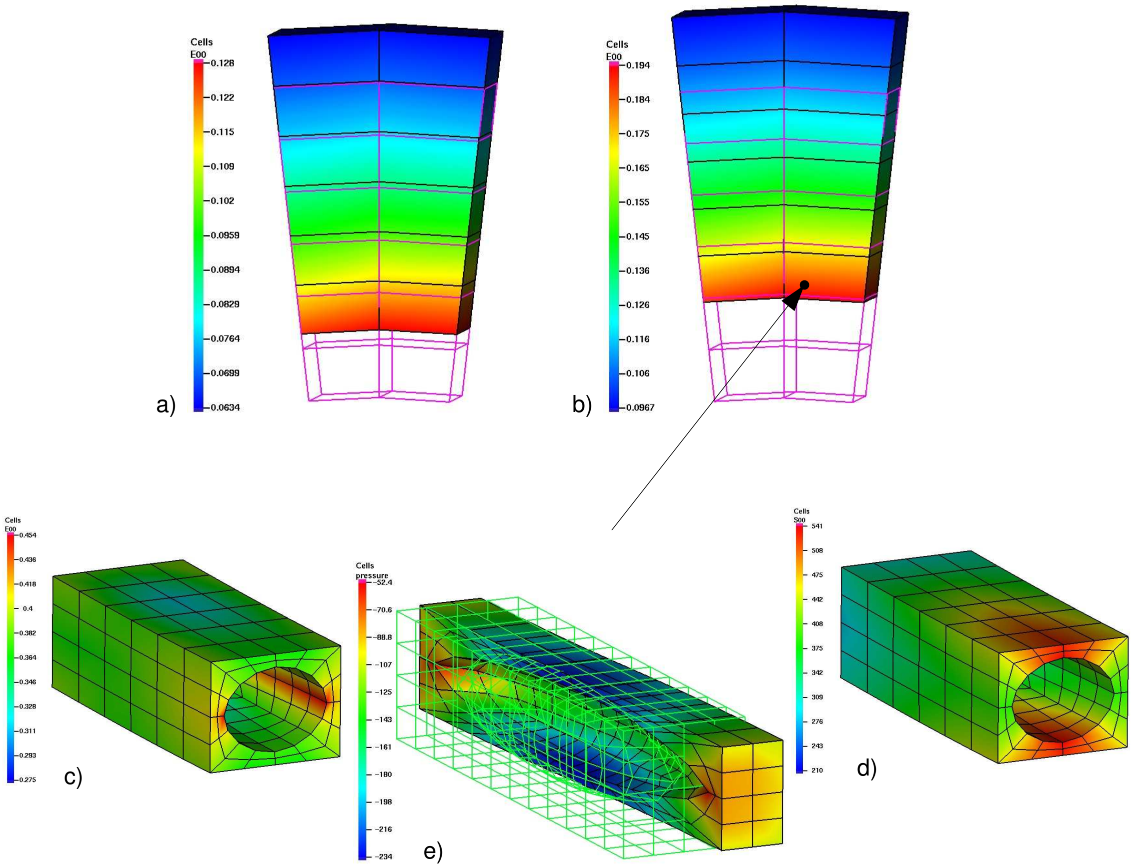

Fig. 5. Macroscopic deformation a), b); deformation c), pressure d), stress e) at the mircoscopic level.

5. CONCLUSIONS

We have presented the two-scale model of the arterial wall which was obtained by homogenization of the linearized subproblem of large deforming solid. The aim of the present contribution is to show viability of the "homogenization" approach which enables to account for influence of the particular geometrical arrangement of the microstructure on the behaviour of the vessel at the global level. The microstructure was constituted as a periodic array of hyperelastic representative material cells containing the incompressible inclusions, to evoke effect of muscle cells. The model of contraction was introduced in terms of truss fitted to the surface of inclusions.

For numerical treatment of the finite deformation problem using the two scale modelling the microscopic boundary value problems must be solved, each one associated with specific "macroscopic" point. In this way the effective homogenized incremental stiffness is recovered in the whole macroscopic model, so that the deformation at the macroscopic scale can be computed. Currently more realistic viscoelastic models are being developed which account for mass redistribution in the extracellular tissue.

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