



BVP in MACRO
 $\nabla \cdot \underline{\underline{\sigma}}_M + \underline{\underline{f}} = \underline{\underline{0}} \quad \forall \underline{\underline{x}}_M \in \Omega_M$
 $\underline{\underline{u}} = \underline{\underline{u}}^0 \text{ on } \Gamma_D \text{ (Dirichlet)}$
 $\underline{\underline{\sigma}}_M \cdot \underline{\underline{n}} = \underline{\underline{t}}^0 \text{ on } \Gamma_N \text{ (Neumann)}$

BVP in MICRO
 $\nabla \cdot \underline{\underline{\sigma}}_m = \underline{\underline{0}} \quad \forall \underline{\underline{x}}_m \in \Omega_m$
 $\underline{\underline{\sigma}}_m = \underline{\underline{C}}(\underline{\underline{x}}) : [\underline{\underline{\epsilon}}(\underline{\underline{u}}) + \underline{\underline{\epsilon}}_M]$
 $\underline{\underline{u}}$ is periodic on Ω_m .
 $\underline{\underline{\sigma}} \cdot \underline{\underline{n}}$ is antiperiodic on Ω_m .

Finally, Solving micro BVP we can obtain $\underline{\underline{\sigma}}(\underline{\underline{x}}_m)$ & $\underline{\underline{\epsilon}}(\underline{\underline{x}}_m)$.

We homogenize them by:

$$\langle \underline{\underline{\sigma}} \rangle = \frac{1}{|\Omega_m|} \int_{\Omega_m} \underline{\underline{\sigma}}(\underline{\underline{x}}_m) d\underline{\underline{x}}_m = \underline{\underline{\bar{\sigma}}}$$

$$\langle \underline{\underline{\epsilon}} \rangle = \frac{1}{|\Omega_m|} \int_{\Omega_m} \underline{\underline{\epsilon}}(\underline{\underline{x}}_m) d\underline{\underline{x}}_m = \underline{\underline{\bar{\epsilon}}}$$

$$\Rightarrow \underline{\underline{\bar{C}}} = \frac{\partial \underline{\underline{\bar{\sigma}}}}{\partial \underline{\underline{\bar{\epsilon}}}}$$

4th order tensor
 or 36 component
 in matrix form.

Upscaling

Algorithm:

Macro.Py
 Loop elm in Elements
 Loop gp in Gauss

$\underline{\underline{\bar{\epsilon}}}_M = \underline{\underline{B}} \cdot \underline{\underline{u}}$ at step m in Newton-Raphson.
 call Micro ($\underline{\underline{\bar{\epsilon}}}_M$) return $\underline{\underline{\bar{\sigma}}}, \underline{\underline{\bar{C}}}$
 calculate $\underline{\underline{K}}_T, \underline{\underline{F}}_{int}, \underline{\underline{F}}_{ext}$ where $\underline{\underline{K}}_T = \frac{\partial \underline{\underline{K}}_{int}}{\partial \underline{\underline{u}}}$